Number Concepts and Special Needs Students: The Power of Ten-Frame Tiles

For more than twenty years, teachers have been using counters and connecting cube "trains" and creating base-10 block models to help students develop number sense, that is, an understanding of number concepts, place value, and computation. Unfortunately, not all students successfully develop an understanding of quantity using these models. This article examines what may be missing in the models we most commonly see, then describes how ten-frame tiles may be a more useful tool for building number understanding for many students.

That a manipulative can be touched does not ensure that it will be effective as a "concrete representation." For many students, a cube train of 10 or a base-10 rod, in which one object stands for ten units, is abstract. They can verify that a "train" or a "rod" shows a quantity of ten only by counting each segment. When the same students use ten-frame tiles, they are able to construct number meanings in more useful ways. **Figure 1** compares two different models for numbers 0–10.

To develop basic number-to-quantity understandings, students benefit most from models that provide a countable, visually distinct model for each number; can be organized in obvious and predictable ways; and provide a clearly defined context of ten. Models such as cube trains, widely used to teach number concepts, do not help students build visually distinct models for each quantity. Models such as base-10 blocks assume facility with proportional reasoning (one object stands for more than one countable unit) that many of our special needs students simply do not possess.

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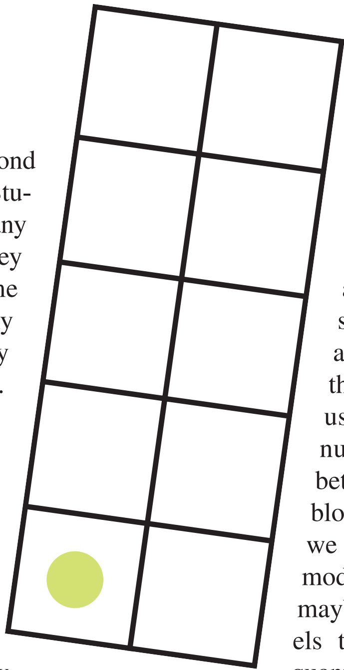
Why Ten-Frame Tiles are Effective

Ten-frame tiles offer special needs students a rich, visual tool for developing understanding of number, place value, and computation. The visual model helps students connect each number name and the quantity it represents. Ten-frame tiles show unique configurations for each numerical quantity from 0 to 10. This helps the visual learner internalize the "shape" of a quantity in instantly recognizable pictures (see **fig. 1**).

Ten-frame tiles reinforce the countability of 10, both as ten units and as a single group of ten. The dots are clearly separate and countable, so students can easily verify the completeness of each ten. Students can easily see the quantity represented without having to recount each time from one.

Figure 2 compares the observations of two mixed groups of second graders (special needs students and students who were not specifically identified). One group used connecting cubes to develop number concepts of 7. The other group used ten-frame tiles. For this activity, the teacher placed a model for a number on the overhead, then turned on



the overhead for one second and turned it off again. Students were asked how many objects—cubes or dots—they had seen. The ten-frame model gave students many ways to conclude accurately that seven dots were shown. The students using the connecting cube model, however, had to count cubes and made many errors in counting. This is not surprising because the human eye cannot discern more than four or five distinct objects in a row without careful counting, unless those objects are set in an information-rich context (Smith 1985, pp. 20–23). Even the student who used a part-part-whole strategy ("4 and 3 more") miscounted. His strategy is excellent; however, the model interfered with his accuracy, which was frustrating.

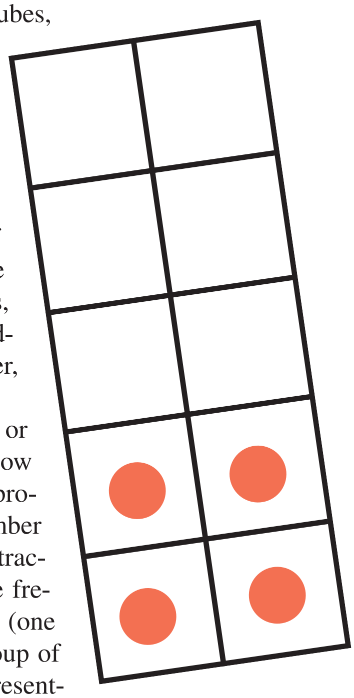
Re-Examining Base-10 Blocks

As soon as students are introduced to numbers greater than twenty, we have them use base-10 blocks. Base-10 blocks assume that students have internalized the concept of tens and ones, and that students understand that one object can stand for more than one countable unit. This is the abstraction of place value. Base-10 blocks are introduced to many students too early, before they have fully internalized the concept of grouping by tens.

Common errors reveal a great deal about number misconceptions. For example, when students are taught to model a two-digit number, such as 27, with base-10 blocks, they often "learn" to put two tens-rods for the first digit and unit cubes for the second digit. Ask the same student to show 27 with ten-frame tiles, however, and you can easily discover whether he or she is actually grouping tens or is merely processing digits without an understanding of the quantity that each digit represents (see **fig. 3**).

What is the first thing your students do when exploring with base-10 blocks? If they are like my students, they build things. The tens-rods make great log cabins. The hundreds-flats make sturdy floors and walls for multi-story structures, and the unit cubes stack into precarious chimneys and fences. A relationship between these "things" and a multiple or power of 10 is the farthest thing from the students' minds.

What do teachers notice when they do the same explorational exercise in staff development sessions? The list of their observations often begins with the geometry of the materials—rectangles, rectangular prisms, cubes, square faces, rectangular faces, right angles—and then the color. The idea that these objects model quantity is usually very low on the list. The numerical/proportional relationships between 1, 10, and 100 inherent in the blocks are rarely mentioned, if at all. If we as teachers do not immediately see models for numbers in base-10 blocks, maybe we should consider using other models that more effectively convey number, quantity, and place value to our students.

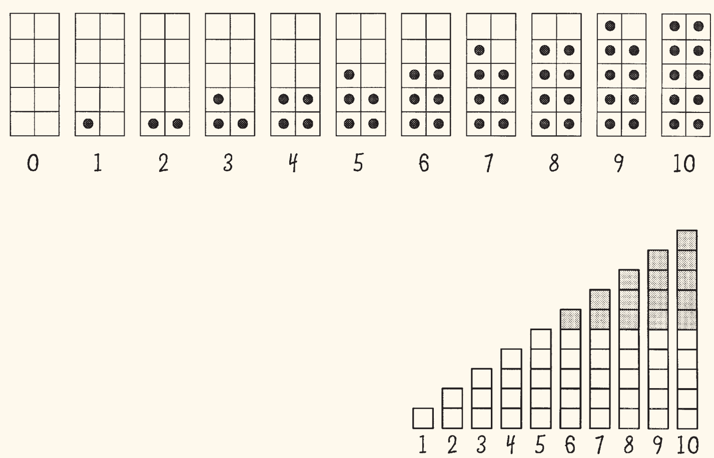


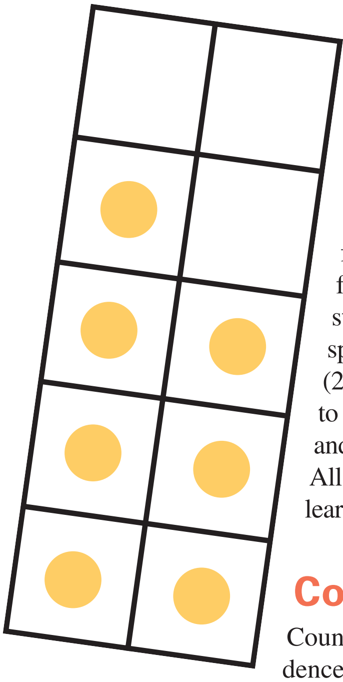
Special needs students often have visual or auditory processing difficulties. Consider how poorly base-10 blocks enhance the visual processing or auditory understanding of number for students who are not ready for the abstractions of place value. Base-10 materials are frequently referred to as *cubes, units,* or *ones* (one unit); *rods* or *tens-rods* (representing a group of ten units); and *flats* or *hundreds-flats* (representing one hundred units). This language can be confusing to many students because only one of each object is visible, making the idea of 10 and the idea of 100 all that more arbitrary.

Figure 1

Comparing ten-frame tiles to cube trains

Ten-frame tiles show a unique picture for each number. By showing each quantity in relation to 10, they provide foundation for place-value concepts. The ten-frame model shows 0. It also shows the attributes of odd and even.





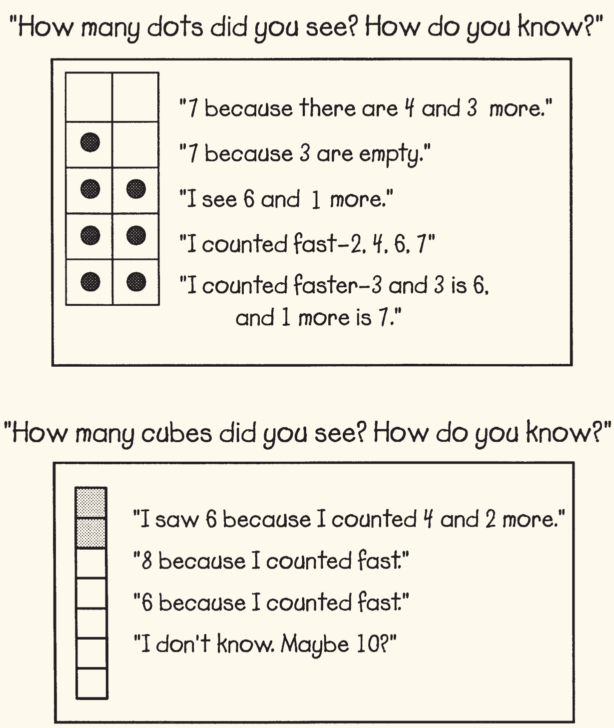
Once students actually understand the concept of groups of ten, any token can stand for that value—a color chip, a tens-rod, or a filled ten-frame tile. Getting to that understanding in the first place is the problem that special needs students face. As Van de Walle (2001) notes, "It is quite common for children to be able to identify the rod as the 'ten' piece and the large square as the 'hundred' piece. . . . All that is known for sure is that they have learned the names for these objects" (p. 31).

Count, Subitize, Compute

Counting is a system of one-to-one correspondence in which each number name relates a quantity to a symbol. It involves associating auditory information (the names of the numbers) with visual information (the written numbers). Subitizing is the "process of instantaneous recognition of number patterns without counting" (Labinowicz 1984, p. 105). Subitizers are able to recognize the number of objects in a group without counting each object. In order to subitize, the student must be able to visualize the quantity in a group, connect it with a numerical value, and verify by means other than counting that the connection is accurate. Cube trains and counters, although useful at the early stages of one-to-one correspondence, do not provide a visually distinct, instantly recognizable picture. Base-10 blocks skip over the subitizing stage of number understanding, especially of quantities less than ten and quantities between tens. Ten-frame tiles, on the other hand, help students develop mental images for numbers that lay a flexible foundation for the intricacies of computation.

Figure 2

Observations of two mixed groups of second graders



Counting and Computation

The leap from counting to computation (arithmetic) is a hurdle for special needs students. Moving beyond counting strategies—counting all, counting up, or counting back—to computation is a challenge for many students. The numerically masterful student composes and decomposes numbers into parts. Activities with ten-frame tiles, in which a countable benchmark for 10 is always visible, give students a model that invites number composition and decomposition as part of visualizing a quantity.

The framework of a visible 10 is essential to developing proficiency beyond counting to computation. **Figure 2** shows how students easily see arithmetic relationships within a number when they analyze the quantities that ten-frame tiles represent. **Figure 4** shows how a student uses that concept in computation. Notice that, with ten-frame tiles, students instinctively look for ways to complete a ten to make computation easier. In this way, they use place-value concepts visually before they generalize place-value ideas conceptually.

Algebra Concepts for the Special Needs Student

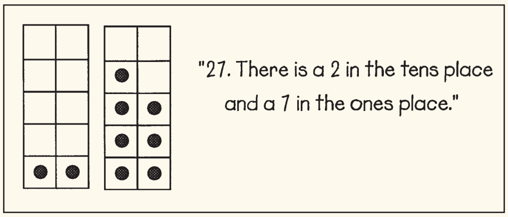
Algebra concepts for elementary-grade students are grounded in four basic understandings (summarized in NCTM 2000, pp. 90–95, 158–62):

- Understand patterns, relations, and functions. "Patterns are a way for young students to recognize order and to organize their world" (p. 91).
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships. "Develop the idea that a mathematical model has both

Figure 3

Ten-frames reveal student understanding.

"What number does the model show? Explain."



Ten-frame tiles provide a useful diagnostic and remediation tool. Errors such as this show that the student does not understand 27 as a quantity.

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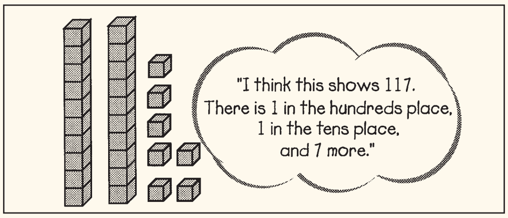
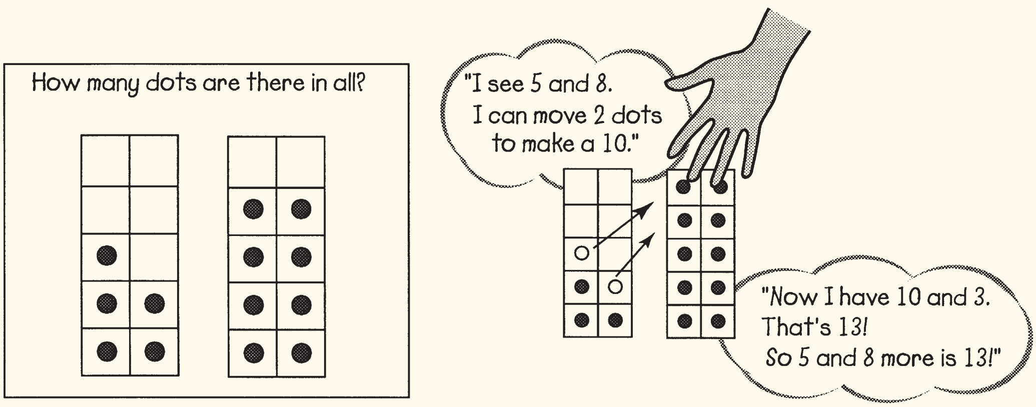


This student understands the concept of place value (hundreds, tens, and ones). His explanation, however, demonstrates that the base-10 blocks do not register as groups of ten. For the visual learner, using a rod to represent 10 may create confusion because the rod looks so much like the numeral 1.

Figure 4

Decomposing and recomposing numbers to add



descriptive and predictive power" (p. 162).

- Analyze change in various contexts.

These understandings are necessary for all students to be successful with algebraic concepts. Special needs students are more likely to develop these understandings when the models that we present for conceptual development take into consideration issues of visual discrimination.

Before students can recognize the organization inherent in our number system, they must see the consistency and organization of numbers related to 10. Ten-frame tiles clearly show the order and organization of numbers in relation to each other and in relation to 10 (see **fig. 1**).

Each ten-frame model is richly descriptive. Consider the ten-frame model for the number 7. Students can count seven dots individually. At the same time, they can see that seven dots are combinations of smaller numbers of dots: 3 + 4; 4 + 3; 6 + 1;

Figure 5

Standard presentation of missing-addend exercise

If a student does not already know basic facts to 8, he or she typically counts up from 3 to find the solution. Students frequently include 3 in their count ("3, 4, 5, 6, 7, 8") and arrive at the wrong solution, 6.

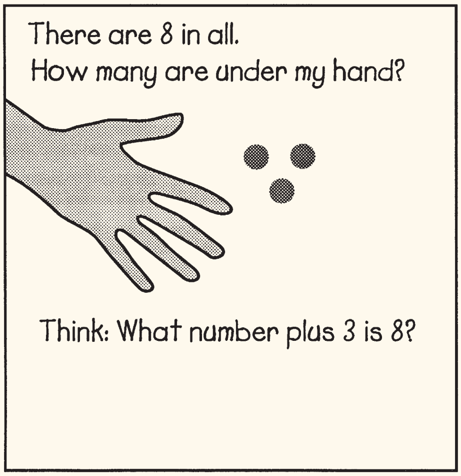
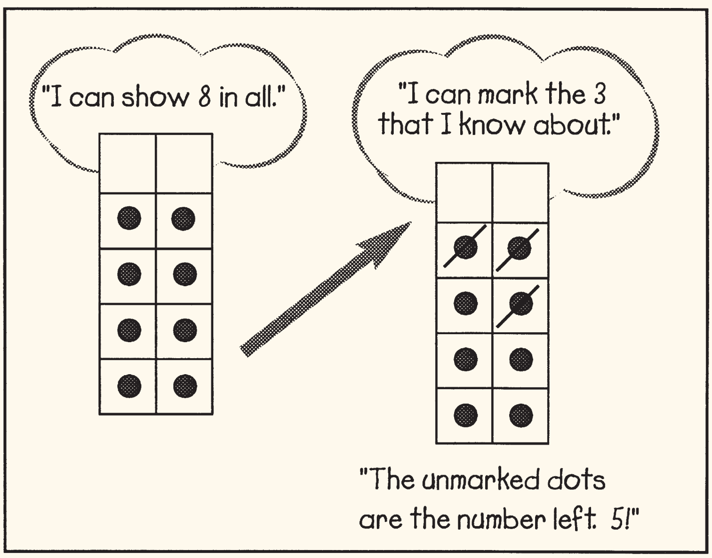


Figure 6

Missing-addends exercise with ten-frame tiles

When students use dot cards to visualize missing addends, they work backward from the known to the unknown. Many use inverse operations as they explain their process (for example, "I'm looking for the number left after I show 3") and find the correct solution by subitizing.



2 + 2 + 2 + 1. They also can immediately see that seven dots are three less than ten dots (see **fig. 2**).

Ten-frame models as a group are richly predictive. Students can easily visualize the outcome after removing one or more dots, the outcome after adding one or more dots, and so on (see **fig. 3**).

The Case of the Missing Addend

Identifying the missing addend in an equation is one specific skill related to number relationships. Most presentations of missing-addend activities require that the student already know basic facts and the principle of inverse operations. Only one part of the typical exercise is modeled concretely. **Figures 5** and **6** compare two different presentations of a missing-addend exercise. **Figure 5** shows the standard presentation, which gives no reliable entry point for a special needs student. **Figure 6** shows the same missing-addends exercise enhanced through the use of ten-frame tiles. Notice the language that the student used when modeling the exercise with the ten-frame tile: She is using the language of subtraction to describe her result!

Ten-frame tiles encourage students to use algebraic intuitions about number relationships in order to reveal the missing addend. Identifying missing addends with the aid of ten-frame tiles invites students to work backward from the known elements (the whole and one of the addends) to the unknown. It allows students to see the whole and the known part in the same image.

After identifying the silhouette of the known part, students easily recognize the silhouette of the missing part. This process of visual discrimination helps students "discover" the inverse relationship between addition and subtraction. When students find missing addends in a context in which they can see the whole and the known part, they "discover" that they can subtract the known from the whole to find the unknown.

Ten-frame tiles can help all students, whether or not they have special needs. Ten-frame tiles help students synthesize visual, auditory, and kinesthetic experiences and develop a solid foundation for counting, place value, and computation. The rich visual patterns of ten-frame tiles reveal to students the underlying algebra of part-part-whole, missing parts, missing addends, expanded notation (10 plus extras), and place value. A special education teacher in Los Angeles Unified School District summed up her experience after replacing base-10 materials with ten-frame tiles in her classroom: "I never imagined that my special needs students would ever get so far! It took about one lesson to show them how the tiles worked. After that, they just took off, and numbers started making sense."

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